# **Math HeadStart Program**

# Workbook





# QUESTION #1 ORDER of OPERATIONS

Consider the following question.  $3 + 4 \times 2$ 

Which procedure gives the correct answer,

add first, then multiply?	or	multiply first, then add			
$3 + 4 \times 2$		$3 + 4 \times 2$			
$= 7 \times 2$		= 3 + 8			
=14		=11			

The correct answer to the math question is 11. We multiply before we add. How do you know to do multiplication before addition? In mathematics, there is a general rule that determines the order in which the basic arithmetic operations of addition, subtraction, multiplication and division are done.

The rule, called the **Order of Operations**, may be stated as follows:

#### **STEPS FOR ORDER OF OPERATIONS**

#### **BEDMAS**

B – brackets, E – exponents, D – division, M- multiplication, A – addition, S – subtraction
Division and multiplication can be done in order of whichever one comes first in the question
Addition and subtraction can be done in order of whichever one comes first in the question

Study the following examples:

#### Example 1

Evaluate  $(3+5) \times 2$ 

#### Solution:

Which operation is done first? The addition comes first because it is inside the brackets. The answer obtained from the bracket is then multiplied by 2.

 $(3+5) \times 2$  Step 1 – Work out the bracket first.  $\downarrow$   $= 8 \times 2$  Step 2 – Multiply. = 16Answer: 16

#### Example 2

Evaluate  $7-15 \div 5 \times 2 + 4$ .

#### Solution:

Note that Step 1 does not apply here because there are no brackets.

$$7 - \underbrace{15 \div 5}_{\checkmark} \times 2 + 4$$

$$= 7 - \underbrace{3 \times 2}_{\checkmark} + 4$$

$$= 7 - 6 + 4$$

$$= 5$$

$$Step 2 - First do all the multiplication and division in order from left to right. Note that the division comes first.
$$Step 3 - Now do the addition and subtraction from left to right. In this case, the subtraction comes first.$$$$

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Answer: 5

#### Example 3

Evaluate  $27 - 2(6+8) \div 4 + 2 - 10$ . Solution:

$27 - 2(6+8) \div 4 + 2 - 10$	Step 1 – First, work out the bracket. Recall that a number placed directly beside a bracket means multiplication.
$= 27 - \frac{2 \times 14}{4} \div 4 + 2 - 10$	Step 2 – Now do the multiplication and division <u>from left to right.</u> In this case, the multiplication comes first.
$=27 - 28 \div 4 + 2 - 10$	
= <u>27-7</u> +2-10	Step 3 – Finally, add and subtract as you go from <u>left to right</u> .
$= \frac{20+2}{4} - 10$	
= 22 - 10 = 12	
Answer: 12	

#### **Exercise 1**

Evaluate the following using the order of operations.

1.	$3 + 4 \times 2$	9.	$(7+3) \div 5 + 2(8-6)$	17.	8-3 (9-2) ÷ 21
2.	$5 - 2 \times 2 + 6$	10.	16 + 4(7 + 8) - 30 + 4	18.	(2+5)(7-3)(4+1)
3.	4 (6 – 2) – 5	11.	$44-4\times5-4\times6$	19.	8 (13 - 7 + 4) - 79
4.	$8\times3-14\div7$	12.	1 + (2 + 3)(4 + 5) + 6	20.	$18 \div 6 \times 10 \div 5 \times 4$
5.	(7+5) ÷ 6-2	13.	11 - 5 + 2 - 3 - 1 + 8		
6.	$20 \div 5 \times 2$	14.	$4 + 12 \times 2 \div 6 \times 3$		
7.	$3(6+3\div 3)+2$	15.	$15 \div (17 - 2) + 6 (12 \div 6 - 2)$		
8.	$15 - 44 \div 11 + 7$	16.	$7 + (8 - 6) \times 0 - 20 \div (2 + 3)$		



Exe	ercise 1 Answers						
1.	11	7.	23	13.	12	19.	1
2.	7	8.	18	14.	16	20.	24
3.	11	9.	6	15.	1		
4.	22	10.	50	16.	3		
5.	0	11.	0	17.	7		
6.	8	12.	52	18.	140		

# QUESTION #2 ARITHMETIC MEAN

You often hear or read statements like:

- The average age of students at Humber College is 24.
- The mean temperature for the month of July was 22 °C.

Average/mean is used to express a value for a given set of numbers.

#### ARITHMETIC MEAN

The **arithmetic mean** (usually just called the mean) is a more formal way of saying the **average**. In most cases, the terms average and mean are interchangeable.

To find the mean value for a list of values, simply **add** up all the values given and **divide** this total by the number of values in the list.

#### Mean = <u>Sum of Values</u> Number of Values

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Example 1

Find the mean age for students having the following ages: 22, 25, 31, 41, 46 and 33.

Solution:

#### Mean = <u>Sum of Values</u> Number of Values

*Step 1 Add the values given to obtain the sum.* 

22 + 25 + 31 + 41 + 46 + 33 = 198

*Step 2* Divide by the number of values added. In this case, six student ages were added.

 $198 \div 6 = 33$ 

Answer: The mean age of the six students is 33.

#### Exercise 2

Find the mean for the following.

- 16, 9, 14, 11, 7, 12, 22
   37, 41, 79, 64, 34
- **3.** 8, 11, 5, 3, 7, 2
- **4.** 301, 207, 263, 267, 239, 265
- **5.** 25, 22, 20, 24, 20, 21, 22

Exercise 2 Answers 1. mean – 13 2. mean – 51 3. mean – 6 4. mean – 257 5. mean – 22

#### #3 & 4

## **ADDITION OF FRACTIONS**

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#### Example 1

Evaluate  $2\frac{7}{8} + 5\frac{1}{6}$ 

#### Solution:

First, add the whole numbers 2 and 5.

$$2\frac{7}{8} + 5\frac{1}{6} = 2 + 5 + \frac{7}{8} + \frac{1}{6}$$
$$= 7 + \frac{7}{8} + \frac{1}{6}$$

Determine the LCD for the denominators 8 and 6.

Now continue with the addition.

 $7 + \frac{7}{8} + \frac{1}{6} = 7 + \frac{21}{24} + \frac{4}{24}$   $= 7 + \frac{25}{24}$   $= 7 + 1\frac{1}{24}$   $= 8 + \frac{1}{24}$   $= 8 + \frac{1}{24}$  Criginal fractions were changed to equivalent fractions with LCD of 24. Fractions were added. Fraction was changed to mixed number. Then the whole numbers were added (7 + 1 = 8) to arrive at the final answer.  $2 + 5 + \frac{1}{6} = 8 + \frac{1}{24}.$ 

The exact same rules hold for subtraction of fractions, the only difference is instead of adding you have to subtract.

Example 2 Evaluate  $\frac{\frac{4}{5}}{\frac{2}{3}}$ Solution:  $\frac{4}{5} \div \frac{2}{3} = \frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$ 

#### Exercise 3

Evaluate (perform the addition). Where possible reduce to lowest terms.

1.	$\frac{5}{9} + \frac{1}{9}$	7.	$\frac{1}{12} + \frac{7}{8} + \frac{2}{3}$
2.	$\frac{7}{12} + \frac{5}{12} + \frac{1}{12}$	8.	$\frac{5}{8} + \frac{3}{20} + \frac{2}{5}$
3.	$\frac{3}{10} + \frac{1}{5}$	9.	$\frac{3}{4} + \frac{7}{10} + \frac{3}{5}$
4.	$\frac{13}{14} + \frac{8}{21}$	10.	$1\frac{3}{4}+2\frac{1}{3}$
5.	$\frac{7}{15} + \frac{9}{10}$	11.	$4\frac{5}{7}+5\frac{11}{14}$
6.	$\frac{7}{10} + \frac{4}{5} + \frac{3}{2}$	12.	$7\frac{3}{10} + 2\frac{1}{12}$

#### Exercise 4

Evaluate the following:

1.  $\frac{\frac{3}{4}}{\frac{2}{7}}$ 2.  $\frac{\frac{2}{5}}{\frac{3}{8}}$ 3.  $\frac{\frac{3}{5}}{\frac{2}{6}}$  **Exercise 3 Answers** 

1. $\frac{2}{3}$	<b>2.</b> $1\frac{1}{12}$	3. $\frac{1}{2}$
4. $1\frac{13}{42}$	5. $1\frac{11}{30}$	<b>6.</b> 3
7. $1\frac{5}{8}$	8. $1\frac{7}{40}$	9. $2\frac{1}{20}$
<b>10.</b> $4\frac{1}{12}$	<b>11.</b> $10\frac{1}{2}$	12. $9\frac{23}{60}$

#### **Exercise 4 Answers**

**1.**  $2\frac{5}{8}$  **2.**  $1\frac{1}{15}$  **3.**  $1\frac{4}{5}$ 



#### #5

# **MULTIPLYING FRACTIONS**

Example 1

Evaluate  $2\frac{7}{8} \times \frac{6}{7}$ 

#### Solution:

Change all fractions to improper form and then multiply:

 $\frac{23}{8} \times \frac{6}{7} = \frac{138}{56}$ 

 $\frac{138}{56} = 2\frac{26}{56} = 2\frac{13}{28}$  (Change the fraction back to mixed format and then reduce the fraction)

#### **Exercise 5**

Evaluate. Express answers in lowest terms.

1.	$\frac{1}{3} \times \frac{4}{5}$	5.	$3\frac{2}{3} \times 7\frac{1}{5}$	9.	$1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 2$
2.	$\frac{5}{6} \times \frac{9}{20}$	6.	$\frac{1}{8} \times 2\frac{2}{3} \times 2\frac{2}{5}$	10.	$3 \times 1 \frac{7}{12} \times 2$
3.	$8 \times \frac{11}{12}$	7.	$1\frac{4}{5} \times 2\frac{1}{7} \times 4\frac{2}{3}$	11.	$3\frac{1}{2} \times 2\frac{3}{4} \times 1\frac{1}{2}$
4.	$3\frac{1}{2} \times 14$	8.	$\frac{11}{17} \times 2\frac{5}{6} \times 3$	12.	$5\frac{2}{15} \times 2\frac{6}{7} \times 3\frac{2}{11}$

#### **Exercise 5 Answers**

1. $\frac{4}{15}$	2. $\frac{3}{8}$	<b>3.</b> $7\frac{1}{3}$	<b>4.</b> 49	<b>5.</b> $26\frac{2}{5}$	6. $\frac{4}{5}$
<b>7.</b> 18	8. $5\frac{1}{2}$	<b>9.</b> 5	<b>10.</b> $9\frac{1}{2}$	<b>11.</b> 14 $\frac{7}{16}$	<b>12.</b> 46 $\frac{2}{3}$

#### #6

# **DECIMALS** $\leftrightarrow$ **FRACTIONS**

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#### FRACTIONS TO DECIMALS

For  $\frac{3}{4}$  the equivalent decimal is 0.75.

For  $\frac{1}{2}$  the equivalent decimal is 0.5.

Two methods are used to change a fraction to its equivalent decimal. Choosing which method to use depends on the fraction.

# **CASE 1** - Changing a fraction to a decimal when the denominator is <u>not</u> a power of **10**. (A power of 10 is 10 or 1000, etc.)

Many people know that  $\frac{1}{2} = 0.5$  or that  $\frac{3}{4} = 0.75$ , but how can you find the decimal equivalent of  $\frac{3}{8}$  or  $\frac{1}{5}$  or  $\frac{7}{16}$ ?

To change a fraction into its equivalent decimal when the denominator is not a power of 10, **divide the denominator into the numerator**.

15.000
24
60
<u>56</u>
40
<u>40</u>
0

#### Example 1

Change the fractions **a**) 
$$\frac{1}{5}$$
 **b**)  $\frac{13}{4}$  and **c**)  $3\frac{7}{16}$  into their equivalent decimals.

Solution:  
a) For 
$$\frac{1}{5}$$
, divide 5 into 1.  
b) For  $\frac{13}{4}$ , divide 4 into 13.  
c)  $3\frac{7}{16} = 3 + \frac{7}{16}$ ,  
Disregard the whole number 3  
for now and divide 16 into 7.  
Answers:  $\frac{1}{5} = 0.2$   
 $\frac{13}{10}$   
 $\frac{3.25}{4\sqrt{13.00}} \longrightarrow \frac{13}{4} = 3.25$   
 $\frac{3.25}{4\sqrt{13.00}} \longrightarrow \frac{13}{4} = 3.25$ 

# **CASE 2** - Changing a fraction to a decimal when the denominator is a power of 10. (A power of 10 is 10 or 1000, etc.)

Study the following examples:

$$\frac{7}{10} = 0.7 \qquad \qquad \frac{35}{100} = 0.35$$
$$\frac{9}{100} = 0.09 \qquad \qquad \frac{734}{10,000} = 0.0734$$

Notice that the number of zeros in the denominator of the fraction is the same as the number of decimal places in the decimal number.

 $\frac{7}{10} = 0.7$   $\frac{35}{100} = 0.35$   $\frac{1}{100} = 0.09$   $\frac{734}{10,000} = 0.0734$   $\frac{1}{100} = 0.09$   $\frac{1}{100} = 0.09$   $\frac{1}{100} = 0.0734$   $\frac{1}{100} = 0.0734$   $\frac{1}{100} = 0.0734$ 

Of course, the equivalent decimals can be found by using the same procedure stated in Case 1. However, moving the decimal point according to the number of zeros in the denominator is a much quicker process.

52.4	0.0734
$\frac{734}{10,000} = 0.0734$	10,000)734.0000
10,000	70000
	34000
	<u>30000</u>
	40000
	40000
	0

In the next two examples, mixed fractions have been changed to their equivalent decimals.

$$2\frac{17}{100} = 2 + \frac{17}{100} = 2.17$$
  
two zeros two decimal places

$$14\frac{3}{1000} = 14 + \frac{3}{1000} = 14.003$$
  
three zeros three decimal places

Exercise 6

Convert each fraction to their decimal equivalent.

1. $\frac{3}{20}$	7. $\frac{5}{32}$	<b>13.</b> $\frac{15}{16}$
<b>2.</b> $\frac{13}{100}$	8. $12\frac{47}{10\ 000}$	<b>14.</b> $\frac{9}{100}$
<b>3.</b> $\frac{9}{40}$	<b>9.</b> $\frac{4}{5}$	<b>15.</b> $1\frac{7}{8}$
<b>4.</b> $\frac{1}{2}$	<b>10.</b> $\frac{15}{32}$	<b>16.</b> $\frac{1}{16}$
<b>5.</b> $\frac{1}{8}$	<b>11.</b> $\frac{19}{100}$	<b>17.</b> $3\frac{11}{25}$
6. $2\frac{3}{4}$	<b>12.</b> $2\frac{3}{10}$	<b>18.</b> $67\frac{12}{1000}$

**Exercise 6** 

<b>1.</b> 0.	.15	<b>2.</b> 0.13	<b>3.</b> 0.225	<b>4.</b> 0.5	<b>5.</b> 0.125	<b>6.</b> 2.75
<b>7.</b> 0.	15625	<b>8.</b> 12.0047	<b>9.</b> 0.8	<b>10.</b> 0.46875	<b>11.</b> 0.19	<b>12.</b> 2.3
13. (	0.9375	<b>14.</b> 0.09	<b>15.</b> 1.875	<b>16.</b> 0.0625	<b>17.</b> 3.44	<b>18.</b> 67.012



# QUESTION #7 **PROBLEM SOLVING**

Solving word problems is a necessary skill for mathematics and for everyday life experiences. To be successful at problem solving it is important that you:

- clearly understand the word problem
- determine the mathematical operations needed
- perform the mathematical operations correctly.

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#### **Example 1**

It cost Dianne \$36.84 for 50 litres of gasoline. Calculate the cost per litre to the nearest cent.

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Solution:

READ	<ul> <li>a) What is being asked for? <i>The cost per litre to the nearest cent.</i></li> <li>b) What information is given? <i>total cost of gas is \$36.84</i> <i>50 litres of gas was purchased</i></li> </ul>
DECIDE	To find the cost per litre, we must <b>divide</b> the total cost by the number of litres
SOLVE	$ \begin{array}{r}     0.7368 \\     50)36.8400 \\     \underline{350} \\     184 \\     \underline{150} \\     340 \\     \underline{300} \\     400 \\     \underline{400} \\     0 \end{array} $ Note that the answer is in dollars per litre (\$0.7368/L). Since \$36.84 is in
	dollars, the answer will be in dollars. There are 100 cents in one dollar. Therefore, mentally multiply $0.7368$ by 100. Move the decimal two places to the right and add the ¢ sign.

 $0.7368 \times 100 = 73.68 \phi$  $\approx$  73.7¢ (rounded to a tenth of a cent)

Note: the symbol  $\cong$  and  $\approx$  means approximately equal to.

#### Exercise 7

Solve the following problems.

- 1. On a business trip, Aldo bought gas twice, paying \$15.50 and \$22.40. His meals cost \$52.68 and he paid \$85.74 for a motel room. What were his total expenses?
- 2. Ruth bought a 3.7 kg beef roast for \$18.98. Calculate, to the nearest cent, the price per kilogram.
- **3.** Harprit went grocery shopping last night. He paid \$3.98 for oranges, \$1.25 for bread, \$3.75 for vegetables, \$12.54 for meat and \$4.59 for laundry detergent. He gave the cashier \$30. How much change did he receive?
- 4. It took 28 litres of gasoline to fill the gas tank in Harry's car. If the cost of gas was 131.5¢ per litre, Find:
  - a) how much Harry had to pay.
  - b) how much change he received if he paid with a \$50 bill.
- 5. To heat her apartment, Terri spent \$84.75 on gas in December, \$120.48 in January, \$104 in February and \$92.08 in March. Find, to the nearest dollar, the cost of gas per month.
- 6. Alisha scored 93, 76, 81, 87, 90 and 77 on six math tests. Calculate her mean (average) score for the six tests.
- 7. The manager of Chicken Villa Restaurant employs nine waiters each earning \$8.75/h, four kitchen helpers at \$7.60/h and three cooks at \$14.25/h. If they all work 7.5 hours a day, what are the total wages paid to these employees each day?
- **8.** Andrew purchased the following items at his local supermarket: 4 cans of tuna at \$1.89 per can, 2.3 kg of cherries at \$4.79 per kg, 3 frozen apple pies at \$6.35 each and 2.1 kg of tomatoes at \$2.29 per kg. How much did all these purchases cost him?

**Exercise 7 Answers** 

1.	\$176.32	2.	\$5.13	3.	\$3.89	4.	a) \$36.82,	b) \$13.18
5.	\$100	6.	84	7.	\$1139.25	8.	\$42.44	

#### #8

# CONVERTING BETWEEN FRACTIONS, DECIMALS AND PERCENTS





#### CHANGING FRACTIONS & DECIMALS TO THEIR EOUIVALENT PERCENTS

To change a fraction or decimal to its equivalent percent do the following:

Step 1 Multiply the fraction or decimal by 100 and simplify.

Step 2 Add on the percent sign.

#### Example 1

Change  $\frac{1}{4}$  to its equivalent percent.

Solution:

Multiply by 100, simplify and add a % sign.

$$\frac{1}{4} \times \frac{100}{1} = \frac{100}{4} = 25\%$$

#### Fractions to Decimals

To change a fraction to a decimal, divide the numerator by the denominator.

$$\frac{5}{8} = 8\overline{\smash{\big)}5.00} = 0.625 \qquad \qquad 2\frac{3}{4} = 2 + 4\overline{\smash{\big)}3.00} = 2 + 0.75 = 2.75$$

#### **Decimals to Fractions**

Do you remember how to change 0.35 to its equivalent fraction?

Place the decimal number without the decimal point (35) in the numerator of the fraction. For the denominator, write the digit 1 plus one zero for every decimal place in the original number. Then, if possible, reduce the fraction to lowest terms.

$0.35 = \frac{35}{100} = \frac{7}{20}  \blacktriangleleft$	— There are two decimal places in 0.35, hence two zeros (100).
$0.225 = \frac{225}{1000} = \frac{9}{40}$	← There are three decimal places in 0.225, hence three zeros (1000).
$5.7 = 5\frac{7}{10}$	There is one decimal place in 5.7, hence just one zero (10).
$0.\bar{6} = \frac{6}{9} = \frac{2}{3}$	Repeating digits have nine (9) in the denominator. One repeating digit – the denominator is 9. Two repeating digits – the denominator is 99. For three repeating digits – the denominator is 999.

Exercise 8							
Change the following fractions to percents by first changing the fraction to a decimal.							
1.	$\frac{1}{4}$	3.	$\frac{4}{9}$	5.	$\frac{13}{16}$	7.	$\frac{7}{2}$
2.	$\frac{11}{10}$	4.	$\frac{3}{8}$	6.	$\frac{9}{5}$	8.	$\frac{2}{3}$

#### **Exercise 8 Answers**

1. 25%	<b>2.</b> 110%	<b>3.</b> 44.4%
<b>4.</b> 37.5%	<b>5.</b> 81.25%	<b>6.</b> 180%
7. 350%	<b>8.</b> 67%	

# QUESTION #9 THE CIRCLE

To measure the perimeter of a square, rectangle or triangle you simply add the lengths of the sides. In some cases, using a formula simplifies the process. The figure below is a circle. A circle is defined as a set of points that are equally distant from the centre of a circle.



#### The perimeter of a circle is called the circumference.

The **diameter** of a circle is the length of a line drawn from one point on the circumference through the centre of the circle to a point on the circumference on the opposite side of the circle.



The **radius** of a circle is the length of a line drawn from the centre of a circle to any point on the circumference.



You may have noticed that *the diameter of a circle is actually made up of two radii* (radii is the plural of radius – we say 1 radius but 2 radii). As well, the radius of a circle is one-half the diameter.

Therefore, if a circle has a radius of 6 cm, its diameter is 12 cm. If another circle has a diameter of 28 in. then its radius is 14 in.

**Diameter (D)** = 
$$2 \times \text{Radius (R)}$$
 or

 $\mathbf{D} = \mathbf{2R}$ 

#### THE CIRCUMFERENCE (PERIMETER) OF A CIRCLE



Diameter = 10 cm Circumference = 31.4 cm



Diameter = 16 cm Circumference = 50.2 cm



Diameter = 23 cm Circumference = 72.3 cm

For the three circles at the bottom of the previous page, the diameter and circumference have been measured. Let's see what happens when, for each circle, we divide the circumference by the diameter.

For circle A	For circle <b>B</b>	For circle C
circumference ÷ diameter	circumference ÷ diameter	circumference ÷ diameter
$31.4 \div 10 = 3.14$	$50.2 \div 16 = 3.1375$	$72.3 \div 23 = 3.143$
	≈ 3.14	≈ 3.14

In fact, if you take the circumference of any circle, whatever its size, and divide by its diameter, the result will always be 3.14 (rounded to hundredths). This number, 3.14, is represented by the Greek letter  $\pi$ , (written **pi** but pronounced "pie").

In actual fact, it is a non-repeating decimal that continues for an indefinite number of decimal places without ending. There are various approximations for the value of  $\pi$ . Your calculator may give it rounded to ten decimal places, 3.141592654. Some books use the approximations, 3.14 (rounded to two decimal places) or 3.142 (rounded to three decimal places). For the calculations that we do in this book, we will use  $\pi = 3.14$ .

 $\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14$ 

Now we can obtain a formula to calculate the circumference of any circle. Since  $\frac{\text{Circumference}}{n} = \pi$ , we can rearrange the formula to read:

Diameter

Circumference(C) =  $\pi \times \text{Diameter}(D)$ 

or

 $\mathbf{C} = \pi \mathbf{D}$ 

Example 1

Find the circumference of a circle with a diameter of 23 cm.

Solution:

*Step 1 Draw a circle and label the diameter.* 



*Step 2* Write down the formula, substitute and calculate the circumference.

$$C = \pi D$$
  
= 3.14×23 cm  
= 72.22 cm  
 $\approx$  72.2 cm

Answer: The circumference of the circle is 72.2 cm.

#### Exercise 9

1. Find the circumference of the following circles. Round answers to the nearest whole number.



#### **Exercise 9 Answers**

**1.** a) 81.68 cm b) 251.33 in c) 326.73 cm

#### #10

# **SOLVING DIRECT PROPORTION PROBLEMS**

If quantities are directly proportional, a change in one quantity can be used to determine a change in the other quantity. We will first discuss **problem solving with quantities that are directly proportional.** 

We know that if quantities form direct proportions, an increase in one quantity will result in an increase in the other quantity, or a decrease in one will result in a decrease in the other.

In problem solving, it is important that we develop an organized procedure. A good problem solving procedure gives good results.

Study the following examples paying close attention to the steps to follow.

#### **Example 1**

A pole 16 m high casts a shadow 20 m long. Find the length of the shadow cast by a building 12 m high.

#### Solution:

- *Step 1 <u>Identify the quantities that are proportional</u>. In this example the two quantities are: the height of the objects and the length of the shadow.*
- *Step 2* <u>Set up a 2 by 2 chart representing the two quantities given and substitute the values into the chart. Let 'x' represent the unknown.</u>

height	of object	length of shadow
pole	16 m	20 m
building	12 m	x m

- *Step 3 <u>Decide if the quantities are directly or inversely proportional</u>. In this example, the quantities are directly proportional. As the height of the object increases, the length of the shadow increases.*
- *Step 4* <u>Set up the proportion and solve</u>. Recall that a proportion is an equality of two ratios. The ratio of the object heights equals the ratio of the length of the shadows.

$$\frac{16}{12} = \frac{20}{x}$$

Now solve the proportion using the procedure learned earlier. Extremes are 16 and x. The means are 12 and 20. Extremes = Means

$$\frac{16x = 12 \times 20}{16x} = \frac{12 \times 20}{16 \times 20}$$
$$x = 15 \text{ m}$$

#### Exercise 10

Solve the following problems involving quantities that are directly proportional.

- 1. Halima completed 3 MatHumber books in 2 months. Assuming that she works at the same rate, how many would she complete in 6 months?
- 2. John travelled 270 km in 3 hours. If he maintains the same speed, how far would he travel in 30 minutes?
- **3.** If, on an automobile trip, you can cover 280 km in 3.5 hours, how long will it take to travel 340 km?
- **4.** Sal used 76 tiles to retile his bathroom floor with an area of 18.2 m<sup>2</sup>. He wants to use the same size tiles for his kitchen floor having an area of 22.8 m<sup>2</sup>. How many tiles will he need?
- 5. Joyce spends \$225 on groceries in a two week period. Calculate, to the nearest dollar, the amount of money she would spend in 45 days. Assume she spends at a constant daily rate.
- 6. Calculate, to the nearest litre, the volume of paint that will be needed to cover 500 m<sup>2</sup> of surface if 12 L of paint will cover 78 m<sup>2</sup>.
- 7. If Judy can read 12 pages of her Nursing Manual in 15 minutes, how many pages can she read in 1.25 hours?
- 8. Sheila drives a transport truck for a living. On her trip from Edmundston to Moncton, a distance of 438 km, her truck uses 106 L of gas. Assuming a similar rate of gas consumption, find how far, to the nearest kilometre, she can travel on 85 L of gas.
- **9.** Tony builds cedar chests. He uses 32 m<sup>2</sup> of cedar to make 12 cedar chests. How many square metres of cedar are needed for 9 chests?
- **10.** On a mid-summer day, Sandro measures the shadow of his 3 m high fir tree at 46 cm. He then measures the shadow of a flagpole at 1.21 m. Calculate the height of the flagpole in metres accurate to two decimal places.
- 11. Mary can read 10 pages in 18 minutes. How long will it take her to read 250 pages?
- **12.** A helicopter pilot found that he had flown a distance of 72 km in 14 minutes. If he continues at the same speed, how far will he fly in three-quarters of an hour?
- **13.** Three centimetre nails are sold by weight. If 26 nails cost \$1.78, how many nails can be purchased for \$5?
- 14. Chantal works for a catering company arranging for food and beverages. At a recent party she found that 13 L of fruit punch were needed for a group of 23 people. At the same rate of consumption, find, to the nearest litre, how much she would need for 16 people?

#### **Exercise 10 Answers**

<b>1.</b> 9 books	<b>2.</b> 45 km	<b>3.</b> 4.25h	<b>4.</b> 96 tiles	<b>5.</b> \$723	<b>6.</b> 77L
<b>7.</b> 60 pages	<b>8.</b> 351 km	<b>9.</b> 24 m <sup>2</sup>	<b>10.</b> 7.89 m	11. 7.5h	<b>12.</b> 231.4 km
<b>13.</b> 73 nails	14. 9 L				

### #11

# **SCIENTIFIC NOTATION**

#### **EXPRESSING LARGE NUMBERS IN SCIENTIFIC NOTATION**

There is a quick way to change 450,000 to its scientific notation equivalent of  $4.5 \times 10^5$ . Recall that when multiplying and dividing by powers of 10 (10, 100, 1000, etc.), you shift the decimal point equal to the number of zeros.

450000 = 450000.	For a whole number, the decimal point (not normally written) is to the right of the number as shown.
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	Move the decimal point to obtain a number greater than 1 and less than 10. In this case we move the decimal point 5 places to the left to obtain 4.5.
$450000=4.5\times10^5$	Since the decimal point moved 5 places to the left, the exponent for the power of 10 is 5.

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In scientific notation, **large** numbers are numbers with a **positive** exponent, while **small** numbers (numbers less than 1) have a **negative** exponent.

#### Example 1

Change  $5.19 \times 10^4$  to decimal notation.

#### Solution:

 $5.19 \times 10^4 = 5.19 \times 10,000$ = 51,900 When multiplying a number by 10,000, the decimal point moves 4 places to the <u>right</u>.

Answer:  $5.19 \times 10^4 = 51,900$ 

Exercise 11	Exercise 11				
Express the fo	llowing in decimal notation.				
1. $2.7 \times 10^{-3}$	6. $2.946 \times 10^4$	<b>11.</b> $0.0065 \times 10^8$			
<b>2.</b> $2.7 \times 10^3$	7. 4.55 $\times 10^2$	<b>12.</b> 7.67×10			
<b>3.</b> $5.07 \times 10^{-6}$	8. $2 \times 10^{-5}$	<b>13.</b> $0.0183 \times 10^{-4}$			
<b>4.</b> $5.07 \times 10^6$	9. $14.5 \times 10^2$	<b>14.</b> $0.0072 \times 10^5$			
5. $8.17 \times 10^{-1}$	<b>10.</b> $662 \times 10^{-7}$	<b>15.</b> $4.973 \times 10^2$			
Exercise 11 Answ	/ers				
<b>1.</b> 0.0027	<b>6.</b> 29 460	<b>11.</b> 650,000			
<b>2.</b> 2700	7. 455	<b>12.</b> 76.7			
<b>3.</b> 0.00000507	<b>8.</b> 0.00002	<b>13.</b> 0.00000183			
<b>4.</b> 5,070,000	<b>9.</b> 1450	<b>14.</b> 720			
<b>5.</b> 0.817	<b>10.</b> 0.0000662	<b>15.</b> 497.3			

# #12 BASIC OPERATIONS WITH SQUARE ROOTS

When addition and subtraction of numbers occur under the square root sign you must;

- first add or subtract the numbers,
- then, take the square root on the answer.



For a sum of square roots or a difference of square roots you must;

- first, take the square root of the numbers,
- then, add or subtract the answers.



For **multiplication and division involving square roots**, no such restrictions apply like those for addition and subtraction. This will be illustrated in the following examples.

$\sqrt{4 \times 9} = \sqrt{36} = 6$	$\sqrt{196 \div 49}$ $= \sqrt{4}$ $= 2$	Multiply or divide the numbers under the square root sign then determine the square root of the answer.
		or
$\sqrt{4 \times 9}$	$\sqrt{196 \div 49}$	Take the numbers under the square root sign
$=\sqrt{4}\times\sqrt{9}$	$=\sqrt{196} \div \sqrt{49}$	and write them as the multiplication or division
$= 2 \times 3$	$=14 \div 7$	of separate square roots. Then evaluate.
= 6	=2	

You can also multiply and divide **individual square roots** and then determine the square root of the answer.

It is often an advantage to do this as individual square root numbers may not be perfect squares as shown in the examples that follow.

$$\begin{array}{c|cccc} \sqrt{4} \times \sqrt{36} & & \sqrt{12} \times \sqrt{3} & & \sqrt{10} \times \sqrt{5} \times \sqrt{2} & & \sqrt{75} \div \sqrt{3} \\ = \sqrt{4 \times 36} & & = \sqrt{12 \times 3} & & = \sqrt{10 \times 5 \times 2} & & = \sqrt{75} \div 3 \\ = \sqrt{144} & & = \sqrt{36} & & = \sqrt{100} & & = \sqrt{25} \\ = 12 & & = 6 & & = 10 & & = 5 \end{array}$$

# All answers can be obtained without the aid of a calculator by using the methods shown previously.

Exercise 12 Without the aid of a calculator, evaluate:					
<b>1.</b> $\sqrt{25} + \sqrt{49}$	$8.  \sqrt{100} \div \sqrt{4}$	<b>15.</b> $\sqrt{27+54}$			
<b>2.</b> $\sqrt{16 \times 4}$	<b>9.</b> $\sqrt{5} \times \sqrt{45}$	<b>16.</b> $\sqrt{8} \times \sqrt{3} \times \sqrt{6}$			
<b>3.</b> $\sqrt{81} \sqrt{121}$	<b>10.</b> $\sqrt{128} \div \sqrt{2}$	<b>17.</b> $\sqrt{9 \times 25}$			
<b>4.</b> $\sqrt{36+64}$	<b>11.</b> $\sqrt{400-144}$	<b>18.</b> $\sqrt{400} \times \sqrt{0.09}$			
<b>5.</b> $\sqrt{225} \div \sqrt{25}$	<b>12.</b> $\sqrt{180 \div 5}$	<b>19.</b> $\sqrt{16.2 \div 0.2}$			
6. $\sqrt{25-9}$	<b>13.</b> $\sqrt{49 \div 0.01}$	<b>20.</b> $\sqrt{99+22}$			
7. $\sqrt{25} - \sqrt{9}$	<b>14.</b> $\sqrt{100} \sqrt{36}$	<b>21.</b> $\sqrt{36} + \sqrt{64}$			

<b>Exercise</b> 1	<b>12</b> Answers				
1.12	<b>2.</b> 8	<b>3.</b> -2	<b>4.</b> 10	<b>5.</b> 3	<b>6.</b> 4
7.2	<b>8.</b> 5	<b>9.</b> 15	<b>10.</b> 8	<b>11.</b> 16	12.6
<b>13.</b> 70	<b>14.</b> 4	15.9	<b>16.</b> 12	<b>17.</b> 15	<b>18.</b> 6
<b>19.</b> 9	<b>20.</b> 11	<b>21.</b> 14			

#### #13

# SUBSTITUTION OF VALUES INTO ALGEBRAIC EXPRESSIONS

In the last section, you learned that, in an algebraic expression, each letter (variable) represents an unknown number. You can find a value for the expression if you know the value of each variable in it.

This process of replacing each variable with its value in an expression and then evaluating is called **substitution**.

To carry out the process of substitution, you:

- replace each variable with a set of brackets containing the value to be substituted;
- follow the rules for Order of Operations to evaluate the resulting order of operations question.

It is important that you **use brackets** when you replace each variable with its value otherwise the resulting expression may not conform to the rules for order of operations.

#### **Example 1**

Evaluate the expression 2a - 3b + 4c when a = 3, b = -4 and c = -2

#### Solution:

2a - 3b + 4c = 2(3) - 3(-4) + 4(-2) = 6 + 12 - 8 = 10 Step 1 - Replace the variables with brackets containing the value of the variable. Step 2 - Evaluate following the Order of Operations.

#### Exercise 13

Evaluate the following expressions when a = 2 and b = -3.

1.	a + b	5.	$a^2 + 2ab + b^2$
2.	$a^2 - b^2$	6.	3(2a+b)(a-3b)
3.	$\frac{a+b}{a-b}$	7.	$\frac{a^2 + b^2}{3ab}$
4.	3a + 4b	8.	$\frac{a}{b} - \frac{b}{a}$
Exerc	ise 13 Answers		
<b>1.</b> -1		5.	1
<b>2.</b> -5		6.	33
<b>3.</b> -1/5	5	7.	-13/18

**4**. -6 **8.** 5/6

# **ADDITION AND SUBTRACTION OF LIKE TERMS**

You should remember from previous lessons that, due to the **distributive law** it is sometimes possible to **factor** a sum of signed numbers. Factoring will be used to add and subtract terms.

#### **Example 1**

Simplify: 5a + 8a + 3a	
Solution:	
5a + 8a + 3a	The variable <b>a</b> in each term is a common factor.
= a(5+8+3)	Remove the common factor from each term and place it in front of the expression. The numerical coefficients remain in the bracket.
= a(16) $= 16a$	Add the numerical coefficients and write the answer in conventional form (16 is placed in front of $\mathbf{a}$ ).

Answer: 5a + 8a + 3a = 16a

In Example 1 above, the addition involved like terms.

Exercise 4.3 Simplify the following expressions. 1. 2p - 14q + 9q - 5p2. -7h - (-4k) + 8h + (-3k) + k3.  $13m^2 + 26m^3 - 8m^2 - 22m^3$ 4. -4d - 7e + 5d + 13e - &e5. -8.1ab - 3.5b + 1.1 - 2ab + 4.6b6.  $-3x^2y^3 + 4x^3y^2 + 7x^2y^3 - x^3y^2$ 

#### **Exercise 14 Answers**

**1.** -3p-5q**2.** h + 2k**3.**  $5m^2 + 4m^3$ **4.** d + 4e **5.** -10.1ab + 1.1b + 1.1**6.**  $4x^2y^3 + 3x^3y^2$ 

# QUESTION #15 LAW OF EXPONENTS FOR DIVISION

Let's consider the division of power question:  $a^7 \div a^3$ 

$a^7 \div a^3 = \frac{a^7}{a^3}$
$=\frac{\underline{a}^{1}\cdot \underline{a}^{1}\cdot \underline{a}^{1}\cdot \underline{a}^{1}}{\underline{a}_{1}\cdot \underline{a}_{1}\cdot \underline{a}_{1}\cdot \underline{a}_{1}}$
$=a^4$

Write each power as a product of factors.

Each factor in the denominator cancels a factor in the numerator.

The four remaining factors are written as a power.

This same answer can be obtained by a different method.

a = 
$$\frac{a^7}{a^3}$$
  
=  $a^{7-3}$   
=  $a^4$   
Subtract the exponent in the denominator from the  
exponent in the numerator. This corresponds to three  
factors in the denominator cancelling three factors in the  
numerator.

This suggests that to **divide powers with the same base you subtract the exponent of the divisor** (the denominator in a fraction) **from the exponent of the dividend** (the numerator in a fraction).

Notice that this is exactly opposite to that of the Law of Exponents for Multiplication where you add the exponents.

#### **Division of Powers**

When dividing powers having the <u>same base</u>, subtract the exponent of the divisor from the exponent of the dividend.

This can be represented algebraically as:

$$A^{m} \div A^{n} = A^{m-n}$$
$$\frac{A^{m}}{A^{n}} = A^{m-n}$$

1.	$\mathbf{x}^6 \div \mathbf{x}^2 = \mathbf{x}^{6-2}$ $= \mathbf{x}^4$	When dividing powers with the same base, subtract the exponents.
2.	$\mathbf{h}^7 \div \mathbf{h} = \mathbf{h}^7 \div \mathbf{h}^1$	<u>Note</u> : h expressed as a power is $h^1$ .
	= h <sup>7-1</sup>	
	= h <sup>6</sup>	

Exercise 15
Simplify the following involving the division of powers.
<b>1.</b> $t^9 \div t^5$
<b>2.</b> $x^{10} \div x^3$
<b>3.</b> $d^2 \div d^{-3}$
4. $\frac{c^3}{c^5}$
5. $a^{-3} \div a^2$
6. $\frac{b}{b^{-5}}^{-3}$

# **Exercise 15 Answers 1.** $t^4$ **2.** $x^7$ **3.** $d^5$ **4.** $c^{-2}$ **5.** $a^{-5}$ **6.** $b^2$

# **POWERS OF A POWER**

#### **POWERS – REVIEW AND PRACTICE**

1.	Multiplication	$a^m \times a^n = a^{m+n}$
2.	Division	$a^m \div a^n = a^{m-n}$ $\frac{a^m}{a^n} = a^{m-n}$
3.	Negative Exponent	$a^{-m} = \frac{1}{a^{m}}$ $\frac{1}{a^{-m}} = a^{m}$
4.	Zero Exponent	$a^0 = 1$
5.	Powers	$(a^m)^n = a^{mn}$
6.	Product	$(ab)^n = a^n b^n$ $(a^x b^y)^n = a^{xn} b^{yn}$

#### Laws of Exponents

In the examples that follow, we will **apply the Laws of Exponents to simplify the question**. All **powers will be expressed with positive exponents** and **all numerical powers will be evaluated**.

#### **Example 1**

Simplify and express all powers with positive exponents:  $(3a^2b)^3$ 

Solution:

 $(3a^{2}b)^{3} = 3^{3}a^{2\times 3}b^{3}$ =  $3^{3}a^{6}b^{3}$ =  $27a^{6}b^{3}$ Multiply the exponent of each power within the bracket with the exponent outside. Evaluate the numerical power.  $3^{3} = 27$ 

Answer:  $(3a^2b)^3 = 27a^6b^3$ 

Exercise 16
Simplify the following:
1. $(5ab^2)^4$
2. $(2a^2b^5)^3$
3. $(5ab^4c)^2$
4. $(-x)^4$
5. $-(-x)^2$
6. $12h^2 - 15h^{-2}$
7. $5(gh)^0 + j^0$

#### **Exercise 16 Answers**

- 1.  $625a^4b^8$ 2.  $8a^6b^{15}$ 3.  $25a^2b^8c^2$ 4.  $x^4$ 5.  $-x^2$ 6.  $12h^2 15/h^2$
- 7. 6

# QUESTION #17 SOLVING SIMPLE EQUATIONS

The equation  $\mathbf{m} + \mathbf{3} = \mathbf{10}$  is only true when  $\mathbf{m} = \mathbf{7}$  since 7 + 3 = 10.  $\mathbf{m} = \mathbf{7}$  is the **solution** to the equation. When you are asked to solve an equation, you are required to find the value of the variable that will make the equation true. The number that is the solution to the equation is also called the **root** of the equation. The number  $\mathbf{7}$  is the root of the equation  $\mathbf{m} + \mathbf{3} = \mathbf{10}$ .

#### Example 3

Solve the following equations by applying the Basic Principles of Equations.

**a)** 5m+8=4m+3 **b)** 2-12x = 5x+14-18x

#### Solution:

a)

5m+8 = 4m+3

In order to arrive at a solution of the form [m = ], we must eliminate 8 from the left side of the equation and 4m from the right side.

5m+8 = 4m+3 5m+8-8 = 4m+3-8 5m = 4m-5	To eliminate 8 from the left side, subtract 8 from <u>both sides</u> and then simplify.
5m = 4m - 5 5m - 4m = 4m - 4m - 5 m = -5 <b>Answer:</b> $m = -5$	To eliminate 4m from the right side, subtract 4m from <u>both sides</u> and then simplify.

b)

2 - 12x = 5x + 14 - 18x

First, simplify the right side of the equation by adding like terms.

2-12x = 5x + 14 - 18x2-12x = 5x - 18x + 142-12x = -13x + 14

In order to arrive at a solution of the form [x =], we must eliminate 2 from the left side of the equation and -13x from the right side.

$$2-12x = -13x + 14$$
  

$$2-2-12x = -13x + 14 - 2$$
  

$$-12x = -13x + 12$$
| To eliminate 2 from the left side, subtract 2  
from both sides and then simplify.

$$-12x = -13x + 12$$
  
-12x + 13x = -13x + 13x + 12  
x = 12  
Answer: x = 12

To eliminate -13x from the right side, add 13x to both sides and then simplify.

#### **Exercise 17** Solve the following equations. 5. 3+2a-6=a-81. 7h+6=6h+152. 9q-5=8q-26. 3y+7-13y=-11y-710+4j=10+3j3.

- 7. m-2-4m = 5-7m-11+3m
- 8. 2(3k+1) = 5k 23

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#### **Exercise 17 Answers**

4.

2 - 9x = 6 - 10x

1.	9	<b>3.</b> 0	<b>5.</b> -5	<b>7.</b> –4
2.	3	<b>4.</b> 4	<b>6.</b> -14	<b>8.</b> -25

#### #18

# **SLOPE AND INTERCEPTS OF A LINE**

#### DETERMINING THE X AND Y INTERCEPTS OF A LINE

Below is a graph of the line 3x + 5y = 15.



Note that the line crosses the x-axis at the point (5, 0) and crosses the y-axis at the point (0, 3).

The point at which the line crosses the x-axis (y = 0) is called the **x-intercept**. The x-intercept for the above line is the point (5, 0), so we can say that "**the x-intercept is 5**". To find the x-intercept of a linear equation, let y = 0 and solve for x.

The point at which the line crosses the y-axis (x = 0) is called the y-intercept. The y-intercept for the above line is the point (0, 3), so we can say that "the y-intercept is 3". To find the y-intercept of a linear equation, let x = 0 and solve for y.

#### Example 1

Find the x and y-intercepts for: a) x + 3y = 6 b) 8x - 7y = 28Solution:

a)

$$x + 3y = 6$$

For <u>x-intercept</u>, let y = 0 and solve for x.

$$x + 3(0) = 6$$
$$x + 0 = 6$$
$$x = 6$$

8x - 7y = 28

The x-intercept is 6. This means that the line crosses the x-axis at the point (6, 0).

x + 3y = 6

For <u>y-intercept</u>, let x = 0 and solve for y.

$$0 + 3y = 6$$
$$y = 2$$

The y-intercept is 2. This means that the line crosses the y-axis at the point (0, 2).

#### Answer: The x-intercept is 6 and the y-intercept is 2.

For <u>x-intercept</u>, let y = 0 and solve for x.

$$8x - 7(0) = 28$$
  
 $8x = 28$   
 $x = 3.5$ 

The x-intercept is 3.5. This means that the line crosses the x-axis at the point (3.5, 0).

8x - 7y = 28

For <u>y-intercept</u>, let x = 0 and solve for y.

$$8(0) - 7y = 28$$
  
 $-7y = 28$   
 $y = -4$ 

The y-intercept is -4. This means that the line crosses the y-axis at the point (0, -4).

Answer: The x-intercept is 3.5 and the y-intercept is -4.

#### **SLOPE OF A LINE**

The slope of a line is an indication of how steep the line is. It is defined as the **rise over the run**. In the graph shown below, **Line A** passes through the points (1, 1) and (5, 4).



Consider the triangle formed using these two points. Starting at point (1, 1) the **run is 4 units** to the right (+4) and the rise is 3 units up (+3).

#### <u>THE "SLOPE Y-INTERCEPT" FORM OF THE LINE $\rightarrow$ y = mx + b</u>

The equation of the line on the graph below is 3x + 2y - 12 = 0.

Since the line passes through the point (0, 6) it has a **y-intercept of 6**.

The line runs up to the left so it has a negative slope. The slope can be determined from the triangle shown.



By changing the equation of the line from 3x + 2y - 12 = 0 to the y = mx + b form, the slope and y-intercept can be readily determined,

$$3x + 2y - 12 = 0$$
  

$$2y = -3x + 12$$
  

$$\frac{2y}{2} = -\frac{3}{2}x + \frac{12}{2}$$
  

$$y = -\frac{3}{2}x + 6$$
 (By comparing  $y = -\frac{3}{2}x + 6$  to the  $y = mx + b$  form,  

$$m = -\frac{3}{2}$$
 (the slope) and  $b = 6$  (the y-intercept).

When a linear equation is written in the y = mx + b form, the value for "m" is the slope of the line and the value for "b" is the y-intercept.

#### Example

Change each equation into the y = mx + b form and state the lines slope and y-intercept.

**a)** x-6y+24=0 **b)** 3x+4y+14=0 **c)** 9x-5y=5 **d)** 4y-11=0

Solution:

a)

$$x - 6y + 24 = 0$$
  

$$-6y = -x - 24$$
  

$$\frac{-6y}{-6} = -\frac{x}{-6} - \frac{24}{-6}$$
  

$$y = \frac{x}{6} + 4$$
  
Note

4y = -3x - 14

 $\frac{4y}{4} = -\frac{3x}{4} - \frac{14}{4}$ 

 $y = -\frac{3}{4}x - 3\frac{1}{2}$ 

Answer: The slope of the line is  $\frac{1}{6}$  and the y-intercept is 4.

**b)** 
$$3x + 4y + 14 = 0$$
  
 $4y = -3$   
 $\frac{4y}{4} = -3$ 

Answer: The slope of the line is  $-\frac{3}{4}$  and the y-intercept is  $-3\frac{1}{2}$ .

c)

$$9x - 5y = 5$$
  

$$-5y = -9x + 5$$
  

$$\frac{-5y}{-5} = -\frac{9x}{-5} + \frac{5}{-5}$$
  

$$y = \frac{9}{5}x - 1$$

Answer: The slope of the line is  $\frac{9}{5}$  and the y-intercept is -1.

d) 
$$4y-11=0$$
  
 $4y=11$   
 $\frac{4y}{4}=\frac{11}{4}$   
 $y=4\frac{3}{4}$   
The equation  $y=4\frac{3}{4}$  can be written as  $y=0x+4\frac{3}{4}$ 

Answer: The slope of the line is 0 and the y-intercept is  $4\frac{3}{4}$ .

#### Exercise 18

Express each equation in the y = mx + b form and state the slope and y-intercept.

- 1. y + 3x = 2
- **2.** 2x 3y = 15
- **3.** 2y-6=0
- 4. 20 = 4x + 5y
- 5.  $\frac{x-y}{2} 1 = 0$

#### **Exercise 18 Answers**

**1.** 
$$y = -3x + 2$$
 **2.**  $y = \frac{2}{3}x - 5$  **3.**  $y = 3$  **4.**  $y = -\frac{4}{5}x + 4$  **5.**  $y = x - 2$ 

#### #19

# **FACTORING TRINOMIALS**

We're now ready to factor trinomials of the form  $ax^2 + bx + c$  where a = 1 and  $a \neq 1$ .

Trinomials of the form  $ax^2 + bx + c$  where a = 1 means that the numerical coefficient of the squared term is 1.

Some examples are: •  $x^2 + 8x + 12$ •  $m^2 + 9m - 10$ 

Trinomials of the form  $ax^2 + bx + c$  where  $a \neq 1$  means that the numerical coefficient of the squared term is a number other than 1.

Some examples are:  $4x^{2} + 20x + 25$  where a = 4  $2t^{2} - t - 3$  where a = 2 $5y^{2} + 11y + 2$  where a = 5

#### **Example 1**

Factor the trinomial  $x^2 - 5x - 24$ .

Solution:



**Step 1** *List all the pairs of factors of the constant term.* 

The constant term is -24.

All the pairs of factors that equal 24 are:  $1 \times 24$ ,  $2 \times 12$ ,  $3 \times 8$  and  $4 \times 6$ .

**Step 2** *Determine the pair of factors that <u>add</u> to obtain the coefficient of the x-term.* 

The factors are -8 and +3 because  $-8 \times 3 = -24$  and -8 + 3 = -5.

**Step 3** *Rewrite the trinomial as a 4 term polynomial.* 

The original x-term -5x is replaced by two x-terms having the numerical coefficients of the two selected factors.

-5x is replaced with -8x + 3x.

The 4 term polynomial is  $x^2 - 8x + 3x - 24$ .

#### (When one term is +ve and the other -ve write the -ve one first.)

**Step 4** *Group the terms of the polynomial 2 by 2 and remove common factors.* 

$$x^{2} - 8x + 3x - 24$$
  
=  $(x^{2} - 8x) + (3x - 24)$   
=  $x(x - 8) + 3(x - 8)$   
=  $(x - 8)(x + 3)$ 

The first two terms and last two terms are bracketed. The common factor  $\mathbf{x}$  is removed from the first bracket and  $\mathbf{3}$  from the second bracket.

The two resulting terms have a common factor of (x - 8). Remove it to complete the factoring.

**Answer:**  $x^2 - 5x - 24 = (x - 8)(x + 3)$ 

#### Exercise 19

Factor the following trinomials.

1. 
$$x^{2} - 4x - 21$$
  
2.  $x^{2} + 4x - 21$   
3.  $x^{2} - 8x - 20$   
4.  $x^{2} + 8x - 20$   
5.  $x^{2} - 6x - 40$   
6.  $x^{2} + 6x - 40$ 

#### **Exercise 19 Answers**

1. $(x-7)(x+3)$	<b>4.</b> (x+10)(x-2)
<b>2.</b> (x +7)(x -3)	<b>5.</b> (x -10)(x+4)
<b>3.</b> (x -10)(x+2)	<b>6.</b> (x+10)(x-4)

#### # 20

# **SOLVING ABSOLUTE VALUE EQUATIONS**

#### **INTRODUCTION**

The **absolute value** of a positive or negative number is its positive value or, to state it another way, its magnitude.

The symbol for absolute value is ||.

|5| = 5 The absolute value of +5 is +5. |-5| = 5 The absolute value of -5 is +5.

#### SOLVING ABSOLUTE VALUE EOUATIONS

When solving a simple equation such as  $|\mathbf{x}| = 7$ , the variable, in this case  $\mathbf{x}$ , has two values, namely, +7 or -7.

For the equation,  $|\mathbf{b}| = 14$ , the solution is  $\mathbf{b} = 14$  or  $\mathbf{b} = -14$ .

Note that |m| = -3 is an impossible equation because the absolute value of a number or variable cannot be negative.

Example 1

Solve |2x+3|=11 and verify the roots.

Solution:

$$|2x+3| = 11$$

$$2x+3 = 11 | 2x+3 = -11$$

$$2x = 11-3 \quad \text{or} \quad 2x = -11-3$$

$$2x = 8 \quad | \quad 2x = -14$$

$$x = 4 \quad | \quad x = -7$$

Verification

For 
$$x = 4$$
:  
 $LS = |2x + 3|$  RS = 11  
 $= |2(4) + 3|$   
 $= |11|$   
 $= 11$   
For  $x = -7$ :  
 $LS = |2x + 3|$  RS = 11  
 $= |2(-7) + 3|$   
 $= |-11|$   
 $= 11$ 

#### Example 2

Solve 2|t|+7=4|t|-5 and verify the roots.

Solution:

2|t|+7=4|t|-5

First solve the equation for t.

$$2|t| + 7 = 4|t| - 5$$
  

$$2|t| - 4|t| = -5 - 7$$
  

$$-2|t| = -12$$
  

$$|t| = 6$$
  

$$t = -6$$
  
or  $t = -6$ 

Verification

For $t = 6$ :		For $t = -6$ :		
LS = 2 t  + 7	RS = 4 t  - 5	LS = 2 t  + 7	RS = 4 t  - 5	
= 2 6  + 7	=4 6 -5	= 2  -6  + 7	=4 -6 -5	
$= 2 \times 6 + 7$	$= 4 \times 6 - 5$	$= 2 \times 6 + 7$	$= 4 \times 6 - 5$	
=19	=19	=19	=19	

#### Exercise 20

Solve the following equations and verify the root(s).

1.	$ \mathbf{t}  = 4$	6.	8 3p  + 6 = 11 3p
2.	b  + 10 = 6	7.	6c+2 =5 6c+2
3.	7  m  - 2 = 19	8.	-6 4h-7 +8=17-3 4h-7
4.	2 x+1  = 9	9.	a  +  a  +  a  +  a  = 6
5.	1 - x  + 7 = 11		

#### **Exercise 20 Answers**

1. 4, -4	<b>5.</b> 5, -3
2. no roots	<b>6.</b> −2/3, 2/3
<b>3.</b> 3, -3	<b>7.</b> −1/3
<b>4.</b> 7/2, -11/2	8. no roots

**9.** 3/2, -3/2