Math Head Start Program

Workbook

HUMBER

QUESTION #1

ORDER of OPERATIONS

Consider the following question. $3 + 4 \times 2$

Which procedure gives the correct answer,

add first, then multiply?	or	multiply first, then add?
$3 + 4 \times 2$ $= 7 \times 2$ $= 14$		$3 + 4 \times 2$ $= 3 + 8$ $= 11$

The correct answer to the math question is 11. We multiply before we add. How do you know to do multiplication before addition? In mathematics, there is a general rule that determines the order in which the basic arithmetic operations of addition, subtraction, multiplication and division are done.

The rule, called the **Order of Operations**, may be stated as follows:

STEPS FOR ORDER OF OPERATIONS

BEDMAS

B – brackets, E – exponents, D – division, M- multiplication, A – addition, S – subtraction

Division and multiplication can be done in order of whichever one comes first in the question

Addition and subtraction can be done in order of whichever one comes first in the question

Study the following examples:

Example 1

Evaluate $(3 + 5) \times 2$

Solution:

Which operation is done first? The addition comes first because it is inside the brackets. The answer obtained from the bracket is then multiplied by 2.

$(3 + 5) \times 2$ *Step 1 – Work out the bracket first.*

↓

$= 8 \times 2$ *Step 2 – Multiply.*

$= 16$

Answer: 16

Example 2

Evaluate $7 - 15 \div 5 \times 2 + 4$.

Solution:

Note that Step 1 does not apply here because there are no brackets.

$7 - \underline{15 \div 5} \times 2 + 4$ — *Step 2 – First do all the multiplication and division in order from left to right. Note that the division comes first.*

$= 7 - \underline{3 \times 2} + 4$

$= 7 - 6 + 4$ — *Step 3 – Now do the addition and subtraction from left to right. In this case, the subtraction comes first.*

$= 5$

Answer: 5

Example 3

Evaluate $27 - 2(6 + 8) \div 4 + 2 - 10$.

Solution:

$27 - 2\underline{(6 + 8)} \div 4 + 2 - 10$ — *Step 1 – First, work out the bracket. Recall that a number placed directly beside a bracket means multiplication.*

$= 27 - \underline{2 \times 14} \div 4 + 2 - 10$ — *Step 2 – Now do the multiplication and division from left to right. In this case, the multiplication comes first.*

$= 27 - \underline{28 \div 4} + 2 - 10$

$= \underline{27 - 7} + 2 - 10$ — *Step 3 – Finally, add and subtract as you go from left to right.*

$= \underline{20 + 2} - 10$

$= 22 - 10$

$= 12$

Answer: 12

Exercise 1

Evaluate the following using the order of operations.

1. $3 + 4 \times 2$
2. $5 - 2 \times 2 + 6$
3. $4\,(6 - 2\,) - 5$
4. $8 \times 3 - 14 \div 7$
5. $(\,7 + 5\,) \div 6 - 2$
6. $20 \div 5 \times 2$
7. $3\,(6 + 3 \div 3\,) + 2$
8. $15 - 44 \div 11 + 7$
9. $(7 + 3) \div 5 + 2\,(8 - 6\,)$
10. $16 + 4\,(7 + 8) - 30 + 4$
11. $44 - 4 \times 5 - 4 \times 6$
12. $1 + (2 + 3\,)(\,4 + 5) + 6$
13. $11 - 5 + 2 - 3 - 1 + 8$
14. $4 + 12 \times 2 \div 6 \times 3$
15. $15 \div (17 - 2\,) + 6\,(12 \div 6 - 2\,)$
16. $7 + (8 - 6) \times 0 - 20 \div (2 + 3)$
17. $8 - 3\,(9 - 2\,) \div 21$
18. $(2 + 5)(7 - 3)(4 + 1)$
19. $8\,(13 - 7 + 4) - 79$
20. $18 \div 6 \times 10 \div 5 \times 4$

Exercise 1 Answers

1. 11
2. 7
3. 11
4. 22
5. 0
6. 8
7. 23
8. 18
9. 6
10. 50
11. 0
12. 52
13. 12
14. 16
15. 1
16. 3
17. 7
18. 140
19. 1
20. 24

QUESTION
#2

ARITHMETIC MEAN

You often hear or read statements like:

- The **average** age of students at Humber College is 24.
- The **mean** temperature for the month of July was 22 ºC.

Average/mean is used to express a value for a given set of numbers.

ARITHMETIC MEAN

The **arithmetic mean** (usually just called the mean) is a more formal way of saying the **average**. In most cases, the terms average and mean are interchangeable.
To find the mean value for a list of values, simply **add** up all the values given and **divide** this total by the number of values in the list.

$$\textbf{Mean} = \frac{\textbf{Sum of Values}}{\textbf{Number of Values}}$$

Example 1

Find the mean age for students having the following ages: 22, 25, 31, 41, 46 and 33.

Solution:

$$\textbf{Mean} = \frac{\textbf{Sum of Values}}{\textbf{Number of Values}}$$

Step 1 *Add the values given to obtain the sum.*

$22 + 25 + 31 + 41 + 46 + 33 = 198$

Step 2 *Divide by the number of values added. In this case, six student ages were added.*

$198 \div 6 = 33$

Answer: The mean age of the six students is 33.

Exercise 2
Find the mean for the following.

1. 16, 9, 14, 11, 7, 12, 22
2. 37, 41, 79, 64, 34
3. 8, 11, 5, 3, 7, 2
4. 301, 207, 263, 267, 239, 265
5. 25, 22, 20, 24, 20, 21, 22

Exercise 2 Answers

1. mean – 13
2. mean – 51
3. mean – 6
4. mean – 257
5. mean – 22

By changing the equation of the line from $3x+2y-12=0$ to the $y=mx+b$ form, the slope and y-intercept can be readily determined,

$$3x+2y-12=0$$
$$2y=-3x+12$$
$$\frac{2y}{2}=-\frac{3}{2}x+\frac{12}{2}$$
$$y=-\frac{3}{2}x+6$$

By comparing $y=-\frac{3}{2}x+6$ to the $y=mx+b$ form,

$m=-\frac{3}{2}$ (the slope) and $b=6$ (the y-intercept).

When a linear equation is written in the $y=mx+b$ form, **the value for "m" is the slope of the line** and **the value for "b" is the y-intercept**.

Example

Change each equation into the $y=mx+b$ form and state the lines slope and y-intercept.

a) $x-6y+24=0$ **b)** $3x+4y+14=0$ **c)** $9x-5y=5$ **d)** $4y-11=0$

Solution:

a)
$$x-6y+24=0$$
$$-6y=-x-24$$
$$\frac{-6y}{-6}=-\frac{x}{-6}-\frac{24}{-6}$$
$$y=\frac{x}{6}+4$$

Note

Answer: The slope of the line is $\frac{1}{6}$ and the y-intercept is 4.

b)
$$3x+4y+14=0$$
$$4y=-3x-14$$
$$\frac{4y}{4}=-\frac{3x}{4}-\frac{14}{4}$$
$$y=-\frac{3}{4}x-3\frac{1}{2}$$

Answer: The slope of the line is $-\frac{3}{4}$ and the y-intercept is $-3\frac{1}{2}$.

c) $$9x - 5y = 5$$
$$-5y = -9x + 5$$
$$\frac{-5y}{-5} = -\frac{9x}{-5} + \frac{5}{-5}$$
$$y = \frac{9}{5}x - 1$$

Answer: The slope of the line is $\frac{9}{5}$ and the y-intercept is −1.

d) $$4y - 11 = 0$$
$$4y = 11$$
$$\frac{4y}{4} = \frac{11}{4}$$
$$y = 4\frac{3}{4}$$

The equation $y = 4\frac{3}{4}$ can be written as $y = 0x + 4\frac{3}{4}$

Answer: The slope of the line is 0 and the y-intercept is $4\frac{3}{4}$.

Exercise 18

Express each equation in the $y = mx + b$ form and state the slope and y-intercept.

1. $y + 3x = 2$

2. $2x - 3y = 15$

3. $2y - 6 = 0$

4. $20 = 4x + 5y$

5. $\frac{x - y}{2} - 1 = 0$

Exercise 18 Answers

1. $y = -3x + 2$ **2.** $y = \frac{2}{3}x - 5$ **3.** $y = 3$ **4.** $y = -\frac{4}{5}x + 4$ **5.** $y = x - 2$

QUESTION

#19

FACTORING TRINOMIALS

We're now ready to factor trinomials of the form $\mathbf{ax^2 + bx + c}$ where $\mathbf{a = 1}$ and $\mathbf{a \neq 1}$.

Trinomials of the form $\mathbf{ax^2 + bx + c}$ where $\mathbf{a = 1}$ means that the numerical coefficient of the squared term is 1.

Some examples are:
- $x^2 + 8x + 12$
- $m^2 + 9m - 10$

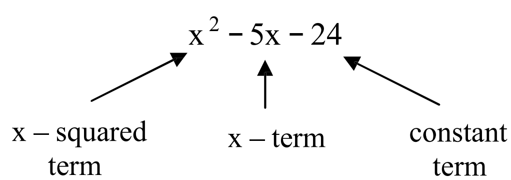
Trinomials of the form $\mathbf{ax^2 + bx + c}$ where $\mathbf{a \neq 1}$ means that the numerical coefficient of the squared term is a number other than 1.

Some examples are:
- $4x^2 + 20x + 25$ where $a = 4$
- $2t^2 - t - 3$ where $a = 2$
- $5y^2 + 11y + 2$ where $a = 5$

Example 1

Factor the trinomial $x^2 - 5x - 24$.

Solution:



Step 1 *List all the pairs of factors of the constant term.*

The constant term is −24.

All the pairs of factors that equal 24 are: 1×24, 2×12, 3×8 and 4×6.

Step 2 *Determine the pair of factors that* <u>*add*</u> *to obtain the coefficient of the x-term.*

The factors are −8 and +3 because $-8 \times 3 = -24$ and $-8 + 3 = -5$.

Step 3 *Rewrite the trinomial as a 4 term polynomial.*

The original x-term $-5x$ is replaced by two x-terms having the numerical coefficients of the two selected factors.

$-5x$ is replaced with $-8x + 3x$.

The 4 term polynomial is $x^2 - 8x + 3x - 24$.

(When one term is +ve and the other –ve write the –ve one first.)

Step 4 *Group the terms of the polynomial 2 by 2 and remove common factors.*

$x^2 - 8x + 3x - 24$

$= (x^2 - 8x) + (3x - 24)$

The first two terms and last two terms are bracketed. The common factor **x** is removed from the first bracket and **3** from the second bracket.

$= x(x-8) + 3(x-8)$

$= \boxed{(x-8)(x+3)}$

The two resulting terms have a common factor of **(x – 8)**. Remove it to complete the factoring.

Answer: $x^2 - 5x - 24 = (\mathbf{x} - \mathbf{8})(\mathbf{x} + \mathbf{3})$

Exercise 19

Factor the following trinomials.

1. $x^2 - 4x - 21$

2. $x^2 + 4x - 21$

3. $x^2 - 8x - 20$

4. $x^2 + 8x - 20$

5. $x^2 - 6x - 40$

6. $x^2 + 6x - 40$

Exercise 19 Answers

1. (x-7)(x+3)

2. (x +7)(x -3)

3. (x -10)(x+2)

4. (x+10)(x-2)

5. (x -10)(x+4)

6. (x+10)(x-4)

QUESTION
20

SOLVING ABSOLUTE VALUE EQUATIONS

INTRODUCTION

The **absolute value** of a positive or negative number is its positive value or, to state it another way, its magnitude.

The symbol for absolute value is $|\ |$.

$|5| = 5$ The absolute value of +5 is +5.
$|-5| = 5$ The absolute value of −5 is +5.

SOLVING ABSOLUTE VALUE EQUATIONS

When solving a simple equation such as $|x| = 7$, the variable, in this case **x**, has two values, namely, $+7$ or -7.

For the equation, $|b| = 14$, the solution is $b = 14$ or $b = -14$.

Note that $|m| = -3$ is an impossible equation because the absolute value of a number or variable cannot be negative.

Example 1

Solve $|2x+3| = 11$ and verify the roots.

Solution:

$$|2x+3| = 11$$

$2x+3=11$	**or**	$2x+3=-11$
$2x = 11-3$		$2x = -11-3$
$2x = 8$		$2x = -14$
$\boxed{x = 4}$		$\boxed{x = -7}$

Verification

For $x = 4$:		For $x = -7$:	
$LS = \|2x+3\|$	$RS = 11$	$LS = \|2x+3\|$	$RS = 11$
$= \|2(4)+3\|$		$= \|2(-7)+3\|$	
$= \|11\|$		$= \|-11\|$	
$= 11$		$= 11$	

Example 2

Solve $2|t| + 7 = 4|t| - 5$ and verify the roots.

Solution:

$$2|t| + 7 = 4|t| - 5$$

First solve the equation for $|t|$.

$$2|t| + 7 = 4|t| - 5$$
$$2|t| - 4|t| = -5 - 7$$
$$-2|t| = -12$$
$$|t| = 6$$

$$\boxed{t = 6} \quad \text{or} \quad \boxed{t = -6}$$

Verification

For $t = 6$:

$LS = 2|t| + 7$
$= 2|6| + 7$
$= 2 \times 6 + 7$
$= 19$

$RS = 4|t| - 5$
$= 4|6| - 5$
$= 4 \times 6 - 5$
$= 19$

For $t = -6$:

$LS = 2|t| + 7$
$= 2|-6| + 7$
$= 2 \times 6 + 7$
$= 19$

$RS = 4|t| - 5$
$= 4|-6| - 5$
$= 4 \times 6 - 5$
$= 19$

Exercise 20

Solve the following equations and verify the root(s).

1. $|t| = 4$

2. $|b| + 10 = 6$

3. $7|m| - 2 = 19$

4. $2|x + 1| = 9$

5. $|1 - x| + 7 = 11$

6. $8|3p| + 6 = 11|3p|$

7. $|6c + 2| = 5|6c + 2|$

8. $-6|4h - 7| + 8 = 17 - 3|4h - 7|$

9. $|a| + |a| + |a| + |a| = 6$

Exercise 20 Answers

1. 4, −4
2. no roots
3. 3, −3
4. 4, −5
5. 5, −3
6. −2/3, 2/3
7. −1/3
8. no roots
9. 3/2, −3/2