TMTH 220
FINAL EXAM FORMULA SHEET

CHAPTER 5: *Graphs*

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} \qquad\qquad y = mx + b$$

CHAPTER 11: *Determinants*

Cramer's Rule:

a) For a system of two linear equations with two unknowns:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

b) For a system of three linear equations with three unknowns:

$$x = \frac{\begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}}{\Delta} \qquad y = \frac{\begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}}{\Delta} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}}{\Delta}$$

where $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$

CHAPTER 12: *Matrices*

Inverse matrix: $AA^{-1} = A^{-1}A = I$

Solution of the system of linear equations: $AX = B$ is $X = A^{-1}B$

Multiplication of matrices: $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} x & u \\ y & v \\ z & w \end{pmatrix} = \begin{pmatrix} ax + by + cz & au + bv + cw \\ dx + ey + fz & du + ev + fw \end{pmatrix}$

CHAPTER 13: *Exponents and Radicals*

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad\qquad (a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad\qquad (a - b)(a + b) = a^2 - b^2$$

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CHAPTER 14: ***Quadratic Equations***

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

CHAPTER 15: ***Oblique Triangles and Vectors***

Law of Sines: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Law of Cosines:

$a^2 = b^2 + c^2 - 2bc\cos(A)$ $\quad \cos(A) = \frac{b^2+c^2-a^2}{2bc}$

$b^2 = a^2 + c^2 - 2ac\cos(B)$ $\quad \cos(B) = \frac{a^2+c^2-b^2}{2ac}$

$c^2 = a^2 + b^2 - 2ab\cos(C)$ $\quad \cos(C) = \frac{a^2+b^2-c^2}{2ab}$

CHAPTER 17: ***Graphs of the Trigonometric Functions***

General Sine Wave: $y = \mathrm{a}\sin(bx + c)$

amplitude $= |\mathrm{a}|$ $\quad$ period $= \frac{360°}{b}$ or $\frac{2\pi}{b}$ $\quad$ frequency $= \frac{b}{360°}$ or $\frac{b}{2\pi}$

phase angle $= c$ $\quad$ phase shift $= -\frac{c}{b}$ $\quad$ $\cos\theta = \sin(\theta + 90°)$

Sine wave as a function of time t: $y = \mathrm{a}\sin(\omega t + \phi)$

amplitude $= |\mathrm{a}|$ $\quad$ angular velocity $= \omega$ $\quad$ period $= \frac{2\pi}{\omega}$

frequency $= \frac{\omega}{2\pi}$ $\quad$ phase angle $= \phi$ $\quad$ phase shift $= -\frac{\phi}{\omega}$

Addition of a sine wave and cosine wave:

$A\sin\omega t + B\cos\omega t = R\sin(\omega t + \phi)$ where

$R = \sqrt{A^2 + B^2}$ and $\phi = \arctan\left(\frac{B}{A}\right)$

Transforming between Polar and Rectangular Coordinates:

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \arctan\left(\frac{y}{x}\right)$$

<u>CHAPTER 18</u>: *Trigonometric Identities and Equations*

$\cot\theta = \frac{1}{tan\theta}$ $\sec\theta = \frac{1}{tan\theta}$ $\csc\theta = \frac{1}{sin\theta}$ $\tan\theta = \frac{sin\theta}{cos\theta}$ $\cot\theta = \frac{cos\theta}{sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2 = \sec^2\theta$ $1 + \cot^2 = \csc^2\theta$

Sum and Difference Identities:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \pm \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A\tan B}$$

Double-Angle Identities:

$$\sin 2A = 2\, sinAcosA$$
$$\cos 2A = cos^2A - sin^2A$$
$$\cos 2A = 1 - 2\, sin^2A$$
$$\cos 2A = 2\, cos^2A - 1$$
$$\tan 2A = \frac{2\tan A}{1 - tan^2 A}$$

Function Values of Special Angles:

θ		**sin** θ	**cos** θ	**tan** θ	**cot** θ	**sec** θ	**csc** θ
30^0	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$	2
45^0	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60^0	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$	2	$\frac{2\sqrt{3}}{3}$ or $\frac{2}{\sqrt{3}}$

CHAPTER 21: *Complex Numbers*

The Imaginary Unit: $j = \sqrt{-1};\ j^2 = -1;\ j^3 = -j;\ j^4 = 1;\ j^5 = j$

Complex Numbers in Polar Form:

a + jb = $r\angle\theta$ (where $r = \sqrt{a^2 + b^2}$ and θ = arctan $(\frac{b}{a})$)

Conversion from Polar to Rectangular Form:

a = r cos θ b = r sin θ

Complex Numbers in Trigonometric Form:

a + jb = r(cos θ + j sin θ) $R\angle(\omega t + \phi) = R\cos(\omega t + \phi) + R\sin(\omega t + \phi)$